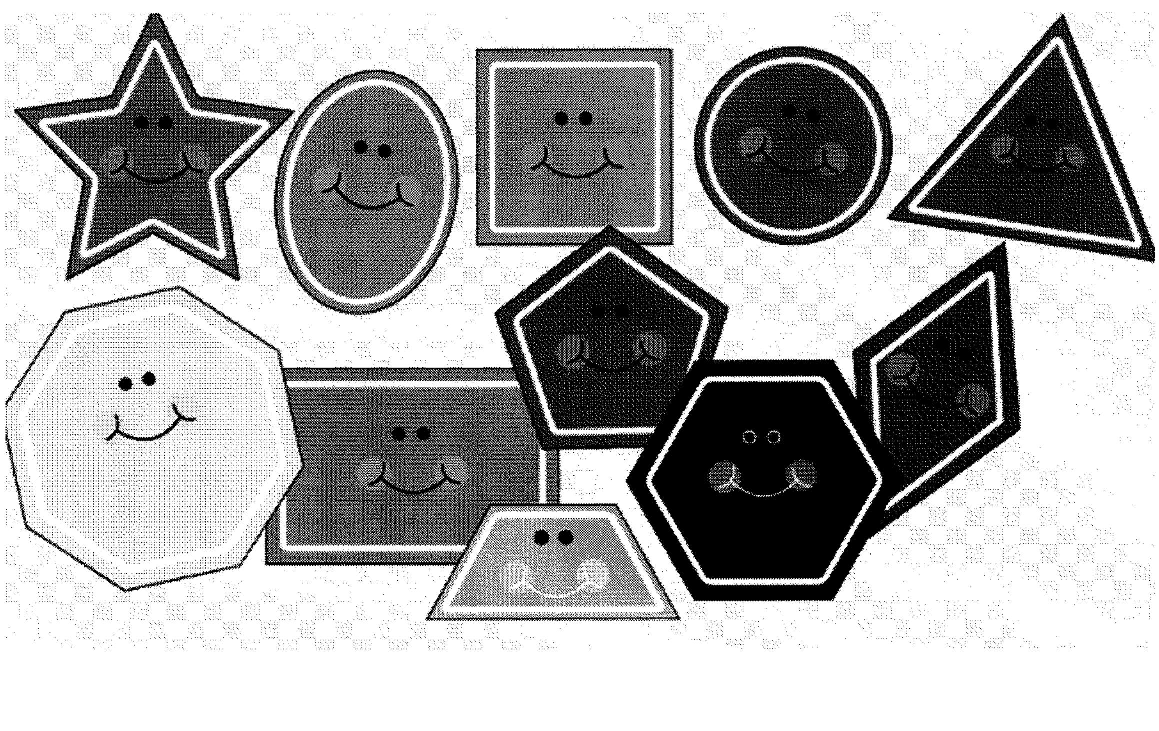
Santee School District

MATHEMATICS PROFESSIONAL DEVELOPMENT

Grades Three and Four



December 6, 2013

MENTAL MATH

During recess there are 98 students on the playground. 87 students join them. How many students are now on the playground?

Record how you mentally solved this problem.

Mathematically Productive Teaching Routine
Structuring Student Math-Talk

Purposes

- Support the development of student-to-student interaction that is consistently equitable, status-free, and mathematically productive
- Provide formative assessment information that drives instructional decisions

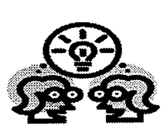
Student Outcomes

- Equitable, status-free, and mathematically productive student-to-student interaction
- Increased metacognitive skills
- Increased capacity to articulate and clarify their math thinking
- Increased math content knowledge
- Improved Mathematical Habits-of-Mind
- Increased accountability and engagement
- Increased self-efficacy as mathematicians

Structures

When students work in a Mathematicians Dyad, Triad, or Quad, the math-talk:

- **Always begins** with "Mathematicians Think Time" (i.e., time to think privately) about the task
- **Always focuses** on each group member's mathematical reasoning, sense making, representations, justifications, and/or generalizations
- **Always ends** with a discussion of ways their ideas are mathematically the same and/or different
- **Always follows** a prescribed structure that provides students "practice" with status-free, and mathematically productive student-to-student interaction

LISTEN & COMPARE 	A. Partner #1 explains her/his ideas while the other partner(s) silently listen to understand Partner #1's thinking. B. When the teacher announces, "*Finish your thought and switch roles*," repeat step A for question/task and student backgrounds. C. (for triads and quads) Repeat until all partners have reported.
REVOICE & COMPARE 	A. Partner #1 speaks while the other partner(s) silently listen to understand Partner #1's mathematical thinking. B. When the teacher announces, "*Finish your thought and Partner #X revoice,*" Partner #X carefully revoices Partner #1's ideas without judging, adapting, or commenting about the correctness or sensibility of the ideas. C. Partner #1 clarifies as needed. D. When the teacher announces, "*Rotate Partners*," Partner #2 speaks while the other partner(s) silently listen to understand. E. When the teacher announces, "*Finish your thought and Partner #Y revoice*," Partner #Y carefully revoices Partner #2's ideas. F. Partner #2 clarifies as needed. G. (for triads and quads) Repeat until all partners have revoiced and reported.
INTERPRET & COMPARE 	A. Two partners exchange their written work for a task. During Private Think Time, the partners study each other's work and, without any discussion, try to understand each other's reasoning. B. Partner #1 reports her interpretation of Partner #2's reasoning. C. Partner #2 clarifies. D. Partner #2 reports his interpretation of Partner #1's reasoning. E. Partner #1 clarifies.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions,

communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

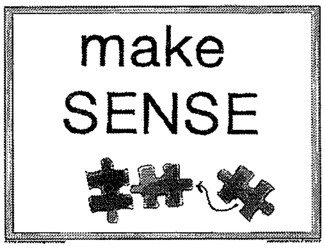
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

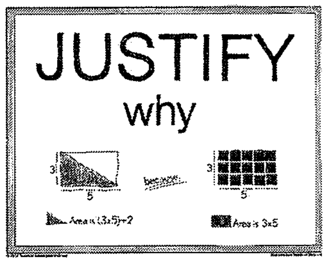
The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

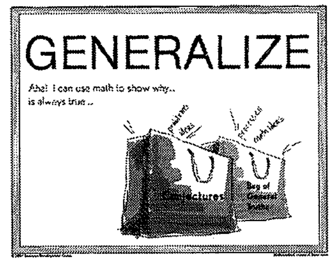
In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Name: **Date:**

STUDENT REFLECTION TOOL: MATHEMATICAL HABITS OF MIND



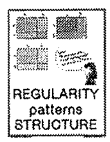
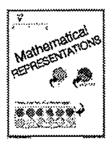
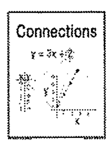
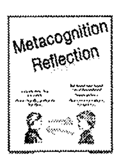




To **make sense** of math ideas and problems, I look for *regularity, patterns, structure, representations, connections, and other math I know*. I *reflect* about my own and others' thinking and *mistakes* and I *persevere* to be sure that ideas and problems make sense.

Math ideas and solutions make sense when I can use *regularity, patterns, structure, representations, connections, and other math I know* to **justify** why the ideas and solutions are always, sometimes, or never true.

I use *regularity, patterns, structure, representations, connections, and other math I know* to make **conjectures** about math ideas I think are always, sometimes, or never true. I create **mathematical generalizations** by justifying why conjectures are valid.

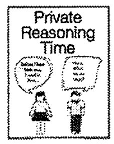
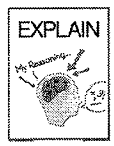
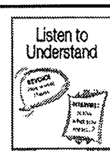
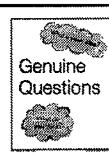
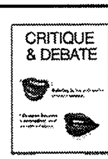
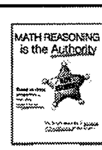
To make **S**ense of math ideas and problems, and to support **C**onjectures, **J**ustifications, and **G**eneralizations:	**S, C, J, G**	**Evidence**
I notice and reason about mathematical REGULARITY in repeated reasoning, PATTERNS, and STRUCTURE (meanings, properties, definitions). 		
I create and reason from MATHEMATICAL REPRESENTATIONS – visual models, graphs, numbers, symbols and equations, and situations. 		
I notice and reason about CONNECTIONS within and across mathematical representations, other math ideas, and everyday life. 		
I explore MISTAKES and STUCK POINTS to start new lines of reasoning and new math learning. 		
I use METACOGNITION and REFLECTION. I think about my math reasoning and disequilibrium – how my thinking is changing and how my ideas compare to other mathematicians' ideas. 		
I PERSEVERE and SEEK MORE. I welcome challenging math problems and ideas, and after I figure something out, I explore new possibilities. 		

Name: **Date:**

STUDENT REFLECTION TOOL: MATHEMATICAL HABITS OF INTERACTION

Use the following scale to rate your use of the Mathematical Habits of Interaction.

0	1	2	3	4
Never/Hardly ever		*Sometimes*		*It's just how I do math!*

When I do math with other mathematicians, we:		Rating	Evidence
Honor each other's right to PRIVATE REASONING TIME before talking about our ideas.			
EXPLAIN how we think and reason mathematically.			
LISTEN TO UNDERSTAND each other's math reasoning about problems, conjectures, justifications, and generalizations.			
Use GENUINE QUESTIONS to inquire about each other's math reasoning about problems, conjectures, justifications, and generalizations.			
Explore MULTIPLE PATHWAYS by applying each other's lines of reasoning.			
COMPARE our math LOGIC and IDEAS to figure out how they are mathematically the same and different.			
CRITIQUE and DEBATE the math logic and truth in each other's reasoning.			
Use MATH REASONING as the AUTHORITY for deciding what is correct and makes sense.			

Hess' Cognitive Rigor Matrix & Curricular Examples: Applying Webb's Depth-of-Knowledge Levels to Bloom's Cognitive Process Dimensions – *M-Sci*

Revised Bloom's Taxonomy	**Webb's DOK Level 1 Recall & Reproduction**	**Webb's DOK Level 2 Skills & Concepts**	**Webb's DOK Level 3** Strategic Thinking/ Reasoning	**Webb's DOK Level 4 Extended Thinking**
Remember Retrieve knowledge from long-term memory, recognize, recall, locate, identify	o Recall, observe, & recognize facts, principles, properties o Recall/ identify conversions among representations or numbers (e.g., customary and metric measures)			
Understand Construct meaning, clarify, paraphrase, represent, translate, illustrate, give examples, classify, categorize, summarize, generalize, infer a logical conclusion (such as from examples given), predict, compare/contrast, match like ideas, explain, construct models	o Evaluate an expression o Locate points on a grid or number on number line o Solve a one-step problem o Represent math relationships in words, pictures, or symbols o Read, write, compare decimals in scientific notation	o Specify and explain relationships (e.g., non-examples/examples; cause-effect) o Make and record observations o Explain steps followed o Summarize results or concepts o Make basic inferences or logical predictions from data/observations o Use models /diagrams to represent or explain mathematical concepts o Make and explain estimates	o Use concepts to solve <u>non-routine</u> problems o Explain, generalize, or connect ideas <u>using supporting evidence</u> o Make <u>and justify</u> conjectures o Explain thinking when more than one response is possible o Explain phenomena in terms of concepts	o Relate mathematical or scientific concepts to other content areas, other domains, or other concepts o Develop generalizations of the results obtained and the strategies used (from investigation or readings) and apply them to new problem situations
Apply Carry out or use a procedure in a given situation; carry out (apply to a familiar task), or use (apply) to an unfamiliar task	o Follow simple procedures (recipe-type directions) o Calculate, measure, apply a rule (e.g., rounding) o Apply algorithm or formula (e.g., area, perimeter) o Solve linear equations o Make conversions among representations or numbers, or within and between customary and metric measures	o Select a procedure according to criteria and perform it o Solve routine problem applying multiple concepts or decision points o Retrieve information from a table, graph, or figure and use it solve a problem requiring multiple steps o Translate between tables, graphs, words, and symbolic notations (e.g., graph data from a table) o Construct models given criteria	o Design investigation for a specific purpose or research question o Conduct a designed investigation o Use concepts to solve non-routine problems o <u>Use & show reasoning, planning, and evidence</u> o Translate between problem & symbolic notation when not a direct translation	o Select or devise approach among many alternatives to solve a problem o Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze Break into constituent parts, determine how parts relate, differentiate between relevant-irrelevant, distinguish, focus, select, organize, outline, find coherence, deconstruct	o Retrieve information from a table or graph to answer a question o Identify whether specific information is contained in graphic representations (e.g., table, graph, T-chart, diagram) o Identify a pattern/trend	o Categorize, classify materials, data, figures based on characteristics o Organize or order data o Compare/ contrast figures or data o Select appropriate graph and organize & display data o Interpret data from a simple graph o Extend a pattern	o Compare information within or across data sets or texts o Analyze and <u>draw conclusions from data, citing evidence</u> o Generalize a pattern o Interpret data from complex graph o Analyze similarities/differences between procedures or solutions	o Analyze multiple sources of evidence o analyze complex/abstract themes o Gather, analyze, and evaluate information
Evaluate Make judgments based on criteria, check, detect inconsistencies or fallacies, judge, critique			o <u>Cite evidence and develop a logical argument</u> for concepts or solutions o Describe, compare, and contrast solution methods o <u>Verify reasonableness of results</u>	o Gather, analyze, & evaluate information to draw conclusions o Apply understanding in a novel way, provide argument or justification for the application
Create Reorganize elements into new patterns/structures, generate, hypothesize, design, plan, construct, produce	o Brainstorm ideas, concepts, or perspectives related to a topic	o Generate conjectures or hypotheses based on observations or prior knowledge and experience	o Synthesize information within one data set, source, or text o Formulate an original problem given a situation o Develop a scientific/mathematical model for a complex situation	o Synthesize information across multiple sources or texts o Design a mathematical model to inform and solve a practical or abstract situation

Student Mathematical Discourse Types

Discourse Types	Examples from Case Study
PROCEDURES/FACTS *No evidence of reasoning.* • Short answer to a direct question • Restating facts/statements/rules • Showing or asking for procedures *Uses meanings, definitions, properties, known math ideas to describe reasoning when:* • Explaining ideas and methods • Questioning to clarify • Noticing relationships/connections • **But *doesn't show why* the ideas/methods work**	
JUSTIFICATION *Reasons with meanings of ideas, definitions, math properties, established generalizations to:* • Show why an idea/solution is true • Refute the validity of an idea • Give mathematical defense for an idea that was challenged	
GENERALIZATION *Reasons with math properties, definitions, meanings of ideas, established generalizations, and mathematical relationships as the basis for:* • Making conjectures about what might happen in the general or special cases Or • Justifying a conjecture about what will happen in the general or special cases	

Switch-arounds for multiplication
Alice
Grade 3, January

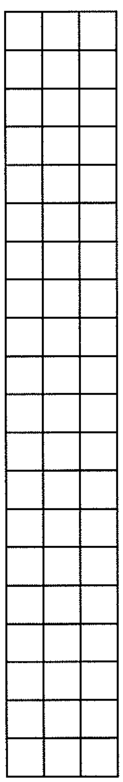
My students had come up with the term *switch-around* in the fall to describe the relationship between 5 + 4 and 4 + 5. Although I mentioned the term *Commutative Property* (I do want them to hear the language of mathematics), they preferred to use their own term. We continued to talk about the switch-around rule.

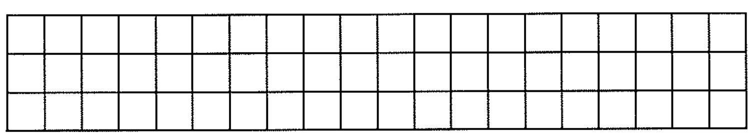
Although in September there were questions about all of the operations, at that time they were looking primarily at addition. Now that we were working with multiplication, I thought they would be interested in reconsidering this idea. I asked them to find a way to convince someone that the switch-around rule works for multiplication. The class worked in pairs and used a variety of models for representing multiplication. After watching them work, I thought about the sequence of presentations for the sharing session. I wanted to have the discussion build in such a way that it would be coherent and sensible. In my mind, I had categorized the models they had used into three kinds: arrays, skip-counting, and grouping.

Three partner groups had developed models based on arrays to show turning the picture of snap-cube configuration would not change the total.

Martha and Mark had drawn rectangles. One was 3 by 20 and the other was 20 by 3. I asked them to share first.

Martha: This shows three groups of 20 and this way it shows twenty groups of 3.





Twenty 3's

three 20's

Teacher: What makes you think this will work for any numbers, not just three groups of 20 and its switch-around?

Mark: Because we tried more, too.

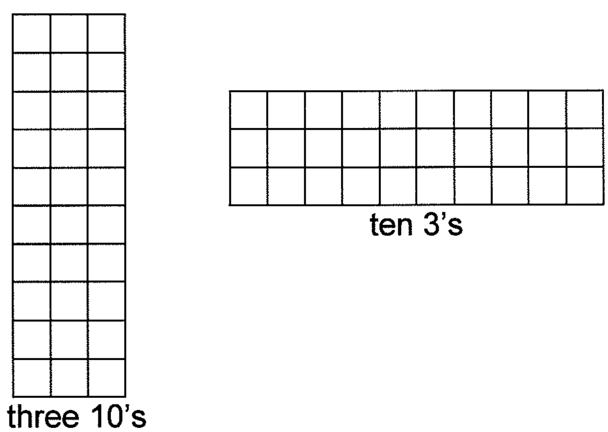
Teacher: Did you try all the numbers?

Martha: No, we didn't have time, but we're really convinced because we really just know because if you have 4 x 6, you'd have the same as 6 x 4, but also it could be a hundred or a thousand or anything. It would have to work.

Susan: I think that's because it's not one of those patterns you have to see all of it to know. I think it's one of those patterns that if you see that it works on most of them, it will work on all of them, and then you just know it's going to work for every single one.

Teacher: Does anyone else have a model that's similar to this one? Marina, how about you and Holly and Mary?

Marina, Holly, and Mary used a layered representation. They built one array as ten sticks of 3 and the other as three sticks of 10, and then put them on top of each other to show how they matched up exactly.



Marina: (Showing their snap-cube construction) This is 3 times 10. We made this layer of ten 3s, and then the layer of three 10 and it fits right on top.

Daniel: The bottom layer is ten 3s and the top layer is three 10s, and they fit.

Marina: It means it works.

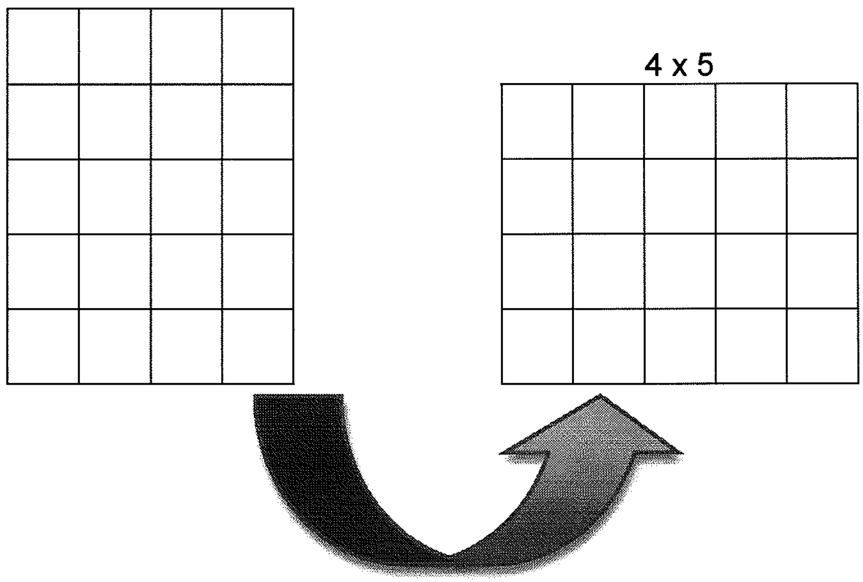
Teacher: How do you know that it works for numbers other than 3 x 10?

Susan: (Again, restating her claim) I think this is just one of those patterns you know works all the time, not like one of our "Theories under Construction" where we might find ones that don't work. But I'm not sure if it works for fractions and negative numbers.

Teacher: Todd and Allan, your model seemed similar to Marina's. Can you show us that next?

Todd: This is a 5 x 4 array, and if you turn it, which is what the arrow shows, you get a 4 x 5. This one is 5 x 4, and it equals 20, and this one is a 4 x 5, and it also equals 20. The 5 from here (pointing to one side of the array) goes here (pointing to the other dimension) and the 4 from here goes to where the 5 was. It's still 20.

Allan: We used the arrow to show turning. We thought of switching as turning.



I was trying to figure out, as they talked, if they noted that their methods would work no matter what the numbers were. However, each time I asked, the students told me about their thinking with specific numbers. I am still wondering about this. I turned to another group.

Teacher: Fran and Daniel, what would you call your model?

Fran: The skip-counting model. We can show how it works if somebody gives us a multiplication problem.

Student: 3 x 12

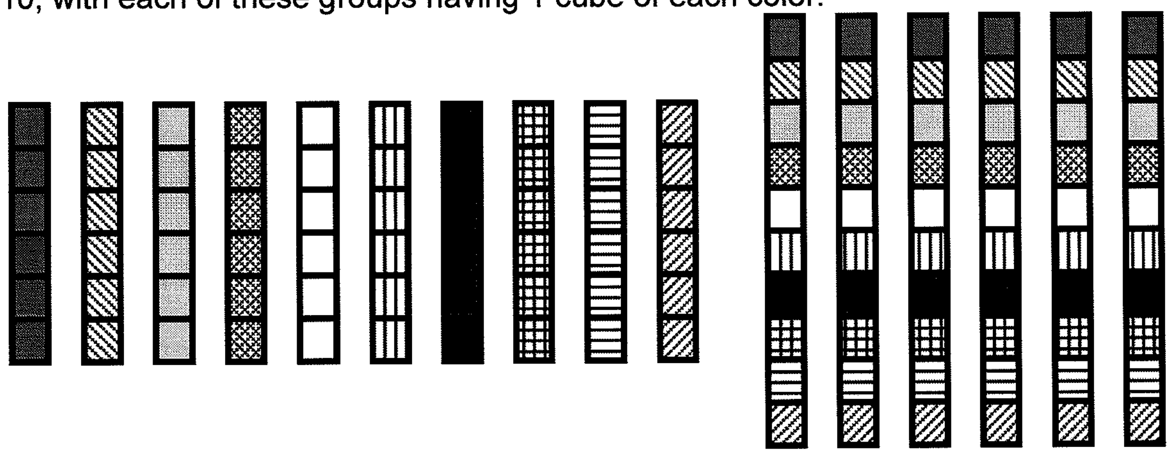
Fran and Daniel: So we do three counts of 12: 12, 24, 36. Then, we do twelve counts of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36. And we get the same answer.

Daniel: So that's how we can use skip-counting. We just do it with the numbers that are in the multiplication sentence. We just do it that number of times.

Teacher: So are you saying that one number becomes the number you're counting by, and the other number is how many counts and then you switch it?

Daniel: Then the second number becomes the number you count by and the other number becomes the number of counts. It has to always work.

The last method for showing the switch-around rule for multiplication was grouping. Sharon and Karen devised this method. They first put the snap-cubes into sticks of six. Each stick was a different color. They made ten of these to show 10 x 6, or ten groups of 6. Then they dealt out 1 cube from each stick, into each of 6 groups. The transformation showed how 10 "six-sticks," each of the same color, became six groups of 10, with each of these groups having 1 cube of each color.



I think their model had great power because it demonstrated what appeared to be a dramatic transformation as the switch happened, going from many sticks of the same color to fewer multicolor groups. It really surprised the class that in spite of the very different look of the "before and after" configurations, nothing of the quantity had been lost or gained. Their surprise indicated to me that this idea of switch-arounds, which they all said they believed, is still being worked on.

As I reflected on this sharing session I began to understand how I could better structure and plan for the learning experiences I want to provide for my students. Asking students to focus on convincing someone, rather than just find number examples, made the level of engagement different and much more inclusive. Everyone, even those who had not yet arrived at a workable model, brought enough of a context to the discussion so as to be able to connect with the ideas being presented. By comparing each other's models to their own, they were able to think about how they were similar and different.

Also, there was a different air of confidence in the arguments my students posed this time. By creating models, they had convinced themselves of something that they were ready to "go public" with. We still need to work on jus how "general" this generalization is. Do their models include the possibility that the numbers might be fractions or very large or even zero? How can we bring up the almost hidden assumptions they might be making about what kinds of numbers we are working with? We will continue to work on these ideas, but this was a good beginning. Through both the creation and presentation of their models and representations, students came to understand more about his property of multiplication and what it means to make a convincing argument.

A question I am still left with is whether the three models are equally convincing. I find the array model offers the most convincing demonstration that multiplication is commutative. No matter what two whole numbers I start with, if the multiplication is represented as an array, I can view it as *n* rows of *m* squares ($n \times m$) or *m* columns of *n* squares ($m \times n$), Do the other representations allow me to picture $n \times m$ and $m \times n$ will always result in the same product?

Name ___________________________

There are 3 boys. Each boy has 2 pockets in his jacket. In each pocket there are 4 candies.
How many candies do the boys have altogether?

How did you get your answer?
Why do you think your answer makes sense?

Name ______________________________

The principal wants to give 6 pencils to each student at the school. There are 598 students at the school. Will 3,000 pencils be enough? How do you know?

Use whatever strategies make sense to you to solve the problem. Use **two or more** different strategies to solve this problem.

Explain how you know your strategies make sense.

6 x 598

598 598 598 598 598 598 3588	54 598 x6 3588	2 x 598 = 1196 2 x 598 = 1196 2 x 598 = 1196 1196 + 1196 + 1196 = 3588
6 x 500 = 3000 6 x 90 = 540 6 x 8 = 48 3000 + 540 + 48 = 3588	500 +90 +8 x 6 6 x 8 = 48 6 x 90 = 540 6 x 500 = 3000 3588	598 x 4 + 2 4 x 8 = 32 4 x 90 = 360 4 x 500 = 2000 2 x 8 = 16 2 x 90 = 180 2 x 500 = 1000 3588
598 x 6 6 x 8 = 48 6 x 90 = 540 6 x 500 = 3000 3588	6 x 600 = 3600 3600 – 12 = 3588	598 x 6 5 x 598 = 2990 1 x 598 = 598 3588

	500	90	8
6	3000	540	48

3000 + 540 + v48 = 3588

Student Discourse Observation Tool

Scripting of Student Discourse	Discourse Type P/F, J, or G

Student Discourse Observation Tool

Scripting of Student Discourse	Discourse Type P/F, J, or G

Classroom Observation - Reflection

1.
What mathematical ideas did students seem to understand? What is your evidence?

2.
With what mathematical ideas were students struggling? What is your evidence?

3.
How would you characterize the students' mathematical discourse?

Commitments

1. Read the article “3 Ways that Promote Student Reasoning.”

2. Work problems 1b and 2b in the article. Mathematically justify your thinking.

3. Conduct a Structured Math Talk with your students a minimum of one time per week.

4. Bring successes and challenges to our next session.